

# SUPERCONDUCTIVITY IN THE BACKGROUND OF TWO-DIMENSIONAL STRIPE SUPERSTRUCTURE

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**Abstract** I propose a superconductivity model, which is based on the assumption that stripes in high- $T_c$  cuprates (a) exist and (b) organize themselves in a two-dimensional superstructure. The model describes hole states, which are localized either inside the stripes or in the antiferromagnetic domains between the stripes. The superconductivity in this model emerges due to the interaction, which is, presumably, mediated by the transverse fluctuations of stripes. The tunnelling density of states obtained from the mean field solution of the model is asymmetric with respect to the chemical potential, has Van Hove singularity identified as a superconducting peak, and, in one of the model regimes, has linear functional form in the vicinity of the chemical potential. The relation between the critical temperature and the zero-temperature superfluid density has “fish-like” form, which quantitatively resembles experimental data. The superconducting order parameter obtained from this model has two components exhibiting non-trivial phase and sign change under translations in real space.

**Keywords:** Stripes, high temperature superconductivity

If energetically *deep* stripes absorbing most of the charge carriers are present in a superconducting (SC) material, then it is likely that the SC mechanism operating in this material would not be operational without stripes. If one further accepts that deep stripes exist in the  $\text{La}_{2-x}\text{Sr}_x\text{CuO}_4$  (LSCO) family of high- $T_c$  cuprates, then it implies that the SC mechanism in LSCO is not operational without stripes. Finally, if one also assumes that the SC mechanism is the same in all families of high- $T_c$  cuprates, then the unavoidable conclusion is that stripes, or, at least strong local inhomogeneities exist in all families of high- $T_c$  cuprates and play a crucial role in the mechanism of superconductivity.

In LSCO, the basic evidence of (dynamic) stripes comes in the form of the well-known four-fold splitting of magnetic  $(\pi, \pi)$  peak, which is observed by inelastic neutron scattering[1], and corroborated by the observation of the elastic response with similar peak pattern in Nd-doped LSCO[2]. The above four-fold splitting has been generally interpreted in the “stripe community” as the

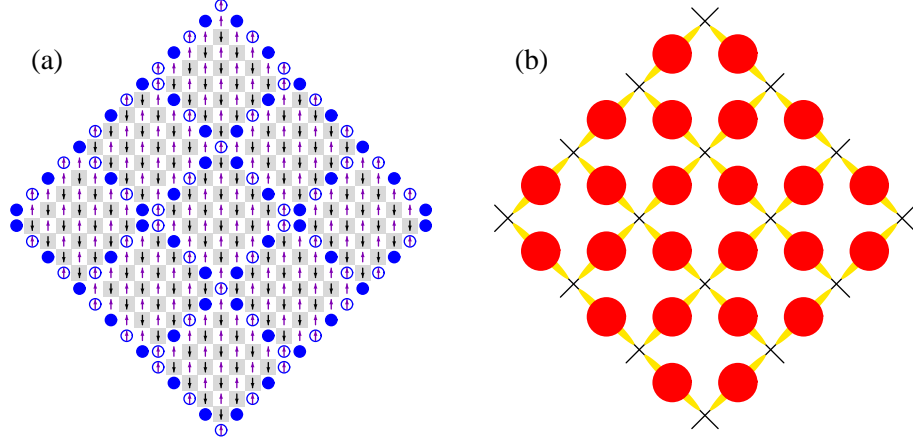


Figure 1. (a) 2D configuration of diagonal stripes. (b) Checkerboard pattern formed by b-states. Circles represent the regions, where b-states are localized.

evidence for two stripe domains, each characterized by a one-dimensional array of stripes running along one of the principal lattice directions. This picture, however, runs into many difficulties, given numerous manifestly two-dimensional (2D) properties of high- $T_c$  cuprates. An alternative interpretation of the four-fold peak pattern, which has been discussed in the literature (see, e.g., Ref. [3]) but never pursued very far, would be based on the 2D arrangement of stripes shown in Fig. 1. The purpose of the present work is to show that the above 2D picture is compatible with superconductivity in general, and with the phenomenology of high- $T_c$  cuprates in particular. This manuscript constitutes a compressed version of a longer paper (Ref. [4]).

The 2D stripe background shown in Fig. 1 implies the existence of two kinds of hole states: a-states — localized inside the antiferromagnetic (AF) domains, and b-states — localized inside the stripes. The on-site energies associated with a- and b-states will be denoted as  $\varepsilon_a$  and  $\varepsilon_b$ , respectively. In underdoped cuprates, the expectation is that  $\varepsilon_a > \varepsilon_b$ . The typical value of the difference  $\varepsilon_a - \varepsilon_b$  should then be identified with the pseudogap. The stripe superstructure should strongly suppress the transport of holes. Therefore, in the zeroth-order approximation, it is reasonable to neglect the direct hopping between both a-states and b-states belonging to different units of the stripe superstructure and also exclude all interaction terms, which shift the center of mass of the hole subsystem.

Here, I introduce, perhaps, the most simple model, which satisfies the above “selection rule”. The model configuration includes one a-state per AF domain, and two b-states per stripe element having opposite orientations of spins.

(Stripe element is a piece of a stripe confined between two subsequent intersections with perpendicular stripes.) The model Hamiltonian is:

$$\mathcal{H} = \varepsilon_a \sum_i a_i^\dagger a_i + \varepsilon_b \sum_{i,j(i),\sigma}^{\eta_i=1} b_{ij,\sigma}^\dagger b_{ij,\sigma} + g \sum_{i,j(i)}^{\eta_i=1} (b_{ij,+}^\dagger b_{ij,-}^\dagger a_i a_j + \text{h. c.}), \quad (1)$$

where single index  $i$  or  $j$  labels AF domains; notation  $j(i)$  implies, that the  $j$ th AF domain is the nearest neighbor of the  $i$ th domain;  $a_i$  is the annihilation operator of a hole inside the  $i$ th AF domain;  $b_{ij,\sigma}$  is the annihilation operator of a hole inside the stripe element separating the  $i$ th and the  $j$ th AF domains;  $\sigma$  is the spin index, which can have two values “+” or “−”;  $\varepsilon_a$  and  $\varepsilon_b$  are position-independent on-site energies for a- and b-states, respectively, counted from the chemical potential; and, finally,  $g$  is the coupling constant. The spin wave function of a-states alternates together with the AF order parameter, i.e. a-states belonging to neighboring AF domains always have opposite spins. The spins of a-states are tracked by index  $\eta_i$ , which can have values 1 or −1. The supercells corresponding to  $\eta_i = 1$  and  $\eta_i = -1$  are to be called “even” and “odd,” respectively. The sum superscript “ $\eta_i = 1$ ” in Eq.(1) indicates that the summation extends only over even supercells. Each transition corresponding to the interaction term in the Hamiltonian (1) can be described either as “two holes from the opposite sides of a given stripe element hopping simultaneously into that element”, or as the reverse process. This kind of interaction is, presumably, mediated by the transverse fluctuations of stripe elements.

The mean-field solution of the above model consists of (i) making the Fourier transform of even  $a$ -operators and odd  $a$ -operators *separately*, which gives, respectively,  $a_e(\mathbf{k})$  and  $a_o(\mathbf{k})$ ; and (ii) introducing the following Bogoliubov transformations:

$$a_e(\mathbf{k}) = u(\mathbf{k})A_e(\mathbf{k}) + v(\mathbf{k})e^{i\phi_a(\mathbf{k})}A_o^\dagger(-\mathbf{k}), \quad (2)$$

$$a_o(-\mathbf{k}) = u(\mathbf{k})A_o(-\mathbf{k}) - v(\mathbf{k})e^{i\phi_a(\mathbf{k})}A_e^\dagger(\mathbf{k}), \quad (3)$$

$$b_{ij+} = sB_{ij+} + we^{i\varphi_{ij}}B_{ij-}^\dagger; \quad (4)$$

$$b_{ij-} = sB_{ij-} - we^{i\varphi_{ij}}B_{ij+}^\dagger, \quad (5)$$

where  $A_e(\mathbf{k})$ ,  $A_o(\mathbf{k})$  and  $B_{ij,\sigma}$  are the annihilation operators of new Bogoliubov quasiparticles;  $\phi_a(\mathbf{k})$  and  $\varphi_{ij}$  are the phases of these transformations; and  $u(\mathbf{k})$ ,  $v(\mathbf{k})$ ,  $s$  and  $w$  are the real numbers obeying the following normalization conditions:  $u^2(\mathbf{k}) + v^2(\mathbf{k}) = 1$ ;  $s^2 + w^2 = 1$ . Phases  $\varphi_{ij} \equiv \varphi(\mathbf{r}_j - \mathbf{r}_i)$  are chosen to be the same for all translationally equivalent stripe elements. Four kinds of translationally non-equivalent stripe elements correspond to four possible even-to-odd nearest neighbor translation vectors  $\mathbf{R}_1 = (1, 1) l/\sqrt{2}$ ;  $\mathbf{R}_2 = (-1, 1) l/\sqrt{2}$ ;  $\mathbf{R}_3 = (-1, -1) l/\sqrt{2}$ ; and  $\mathbf{R}_4 = (1, -1) l/\sqrt{2}$ . (Here  $l$

is the length of a stripe element.) Correspondingly, there exist four independent phases  $\varphi_\alpha = \varphi(\mathbf{R}_\alpha)$ .

Below I consider two most promising cases: Case I — characterized by  $\varepsilon_b = 0$ , and Case II — characterized by  $\varepsilon_a = 0$ . In Case I, the standard variational scheme leads to the following equations for the critical temperature ( $T_c$ ):

$$T_c = \frac{g^2 \left[ \exp \left( \frac{|\varepsilon_a|}{T_c} \right) - 1 \right]}{8|\varepsilon_a| \left[ \exp \left( \frac{|\varepsilon_a|}{T_c} \right) + 1 \right]}, \quad (6)$$

and for the zero-temperature energies of A- and B- quasiparticles :

$$\varepsilon_A(\mathbf{k}) = \sqrt{\varepsilon_a^2 + \frac{1}{4}g^2|V(\mathbf{k})|^2}, \quad (7)$$

$$\varepsilon_B = \frac{g^2}{8N} \sum_{\mathbf{k}} \frac{|V(\mathbf{k})|^2}{\varepsilon_A(\mathbf{k})}, \quad (8)$$

where  $N$  is the total number of supercells, and  $V(\mathbf{k}) = \sum_{\alpha} e^{-i\varphi_\alpha - i\mathbf{k}\mathbf{R}_\alpha}$ . The four phases  $\varphi_\alpha$  are only constrained by condition  $(\varphi_2 + \varphi_4 - \varphi_1 - \varphi_3)/2 = \pi/2 + \pi n$ , where  $n$  is an integer number. The density of B-states thus consists of two symmetric  $\delta$ -function peaks located at  $\pm\varepsilon_B$ . The density of A-states, is continuous but asymmetric with respect to the chemical potential. It has Van Hove singularities at  $\varepsilon_{A0} = \pm\sqrt{\varepsilon_a^2 + g^2}$ , and a gap extending between  $-\varepsilon_a$  and  $\varepsilon_a$ .

In Case II, the analogous results are:

$$T_c = \frac{g^2 \left[ \exp \left( \frac{|\varepsilon_b|}{T_c} \right) - 1 \right]}{8|\varepsilon_b| \left[ \exp \left( \frac{|\varepsilon_b|}{T_c} \right) + 1 \right]}. \quad (9)$$

$$\varepsilon_A(\mathbf{k}) = \frac{g^2 |V(\mathbf{k})| C_{a0}}{8\varepsilon_B}, \quad (10)$$

$$\varepsilon_B = \sqrt{\varepsilon_b^2 + g^2 C_{a0}^2/16}, \quad (11)$$

where  $V(\mathbf{k})$  is the same as in Case I, and  $C_{a0} \equiv \frac{1}{N} \sum_{\mathbf{k}} |V(\mathbf{k})| = 0.958\dots$ . In this case, the density of A-states is symmetric and has Van Hove singularities at  $\varepsilon_{A0} = \pm \frac{g^2 C_{a0}}{4\varepsilon_B}$ , while the  $\delta$ -peaks, corresponding to B-states are asymmetric. Unlike the result for Case I, the density of A-states in Case II has no gap around the chemical potential. Instead, it equals zero at the chemical potential and then increases linearly.

In order to interpret the experimental tunnelling data, it is necessary to assume, that the observed spectra are those of A-states, which means that  $\varepsilon_{A0}$

corresponds to the energy of the experimentally observed SC peak. The B-states are, perhaps, more difficult to observe. However, B-states (in the SC state), or b-states (in the normal state) form a checkerboard pattern shown in Fig. 1(b). Therefore, they may be responsible for the checkerboard patterns seen by scanning tunnelling microscopy[5–7].

The superfluid properties of this model are unusual because of the unusual form of the current operator. Fundamental to this model is the internal current operator, which describes the particle flow between a- and b-states. For the  $i$ th supercell, the internal current operator can be obtained as follows:

$$J_{ab(i)} \equiv -\frac{d}{dt}(a_i^\dagger a_i) = -\frac{ig}{\hbar} \sum_{j(i)} (b_{ij,+}^\dagger b_{ij,-}^\dagger a_i a_j - \text{h.c.}). \quad (12)$$

Operator (12) sums over four possible transitions, each transferring a hole from  $i$ th AF domain to one of the four surrounding stripe elements. When the direction of each of the above transitions is taken into account, the following expression for the translational current operator can be obtained:

$$\mathbf{J}_i^t = -\frac{ig}{2\hbar} \sum_{j(i)} \hat{\mathbf{n}}_{ij} (b_{ij,+}^\dagger b_{ij,-}^\dagger a_i a_j - \text{h.c.}), \quad (13)$$

where  $\hat{\mathbf{n}}_{ij}$  is the unit vector in the direction from the  $i$ th to the  $j$ th supercell.

The internal *supercurrent* corresponding to operator (12) emerges, when the SC solution is modified by adding an extra phase  $\phi_{ab}$  to  $\varphi_{ij}$  in the Bogoliubov transformation for b-states. If the phase  $\phi_{ab}$  is the same for all stripe elements, then the translational supercurrent equals zero. However, when  $\phi_{ab}$  has a weak position dependence, the zero-temperature density of translational supercurrent can be expressed as:

$$\mathbf{j} = \frac{e}{l z_0} \langle \mathbf{J}_i^t \rangle = S_\phi \nabla \phi_{ab}, \quad (14)$$

where, in Case I,

$$S_\phi = \frac{eg^2}{16N\hbar z_0} \sum_{\mathbf{k}} \frac{|V(\mathbf{k})|^2}{\varepsilon_A(\mathbf{k})}; \quad (15)$$

and, in Case II,

$$S_\phi = \frac{eg^2 C_{a0}^2}{32 \hbar z_0 \varepsilon_B}. \quad (16)$$

Here,  $z_0$  is the transverse distance per one SC plane, and  $S_\phi$  is the SC phase stiffness (frequently referred to as superfluid density).

As the doping concentration changes, the value of  $\varepsilon_a - \varepsilon_b$  (characterizing the pseudogap) should, in relative terms, change stronger than the coupling constant  $g$ . Therefore, an approximate relation between  $T_c$  and  $S_\phi$  within one

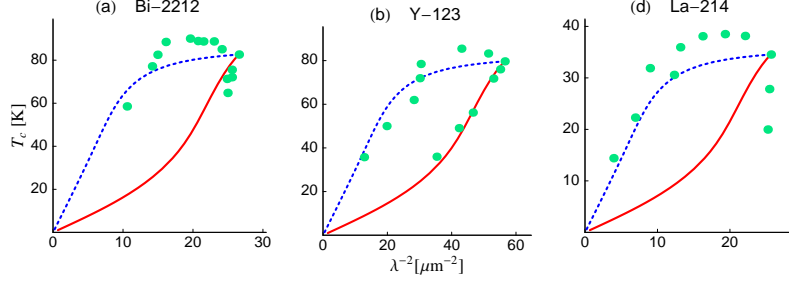


Figure 2.  $T_c$  vs.  $S_\phi$  (presented as  $1/\lambda^2$ , where  $\lambda$  is the in-plane penetration depth of magnetic field): solid line — Case I; dashed line — Case II, circles — experimental points from Ref. [8]. Each plot contains the same pair of theoretical curves, which, initially, were calculated in arbitrary units as described in the text, and then rescaled in such a way that the experimental critical points (evident in every plot) match the theoretical critical point corresponding to  $\varepsilon_a = \varepsilon_b = 0$ .

family of high- $T_c$  cuprates can be obtained by fixing the value of  $g$  and then calculating  $T_c$  and  $S_\phi$  as functions of  $\varepsilon_a$  (in Case I), or  $\varepsilon_b$  (in Case II). The resulting theoretical relation is compared with experiments in Fig. 2.

In conclusion, I have shown that superconductivity is compatible with 2D stripe superstructure. The specific model presented in this work has the following qualitative features resembling the phenomenology of high- $T_c$  cuprates: (i) emergence of the quasiparticle coherence in  $k$ -space only at temperatures below  $T_c$  (see  $\varepsilon_A(k)$ ); (ii) linear density of states in the vicinity of the chemical potential (in Case II); (iii) asymmetry in the tunnelling characteristics; (iv) Van Hove singularity in the tunnelling density of states ( $\varepsilon_{A0}$ ); (v) real space checkerboard pattern in the density of states; (vi) low superfluid density having universal “fish-like” dependence on  $T_c$ . Although not discussed in this paper, the Bogoliubov transformations (2-5) imply a very unconventional symmetry of the SC order parameter, which, in particular, includes the sign change of at least one of the two SC components under translations in real space [4]. A similar prediction has also been made by Ashkenazi in Ref. [9].

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